Chapter 2: Fundamentals

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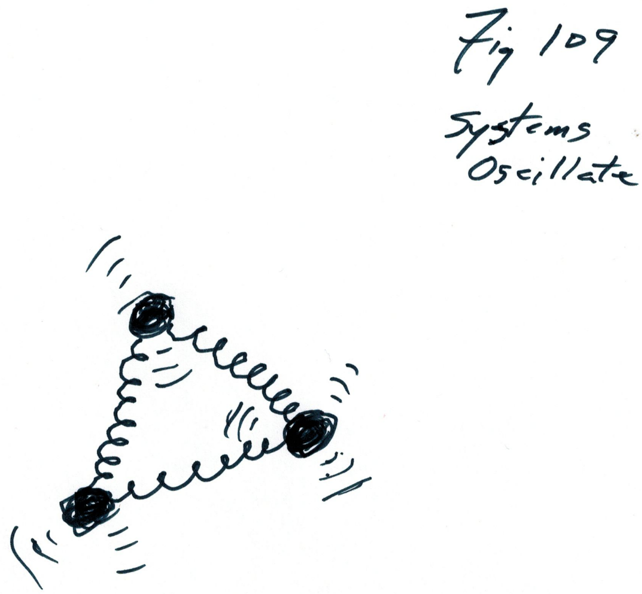
2.1 Definition of system

At its most abstract level a system is comprised of a plurality of parts linked so the behavior of one part affects the behavior of the others either directly or indirectly, either strongly or weakly. To be more accurate a system is a sub-set of parts in the environment that are linked strongly enough to be a recognizable entity and behave as such within the larger environment. In some cases like a system in the form of a living human being its clear which parts are included and which aren't. In an economic system it becomes debatable which parts to include since one system like the auto industry blends into another like the steel industry. Fluids and gases can also form systems. More on that later.

Figure 109 illustrates a simple isolated system using springs to connect the parts, which have mass like little steel balls. A frequent systems behavior is oscillation as the figure attempts to illustrate. This little diagram is a very abstract representation of a system but one to keep in mind. Certainly it applies to systems comprised of discrete parts. However the parts in gaseous systems like the atmosphere or fluidic like the oceans are not so easy to visualize in this manner. Sometimes its necessary to consider a portion of such systems, such as an "blob" of hot air or water, as a part and watch how it moves or changes.



Conserving versus dissipative systems: Conserving systems are systems whose total internal energy remains constant even though the energy associated with each part may be oscillating. That energy is input to the system by some initial outside force that stretches the spring-like bonds away from their equilibrium length. Dissipative systems have a complex technical definition https://en.wikipedia.org/wiki/Dissipative_system but I see them as systems where energy is dissipated by friction or radiation until the system comes to a motionless halt unless they are kept in motion by continued or occasional inputs of energy. A frictionless clock pendulum is a conserving system whose pendulum will swing forever. In the real world it's a dissipative system because friction drains energy while a spring or weight inserts enough –in the form of escapement pushes- to keep it moving. Pushing a swing is the same idea. All real world systems are dissipative, some more than others. The solar system is closest to a conserving real-world system while weather systems like thunderstorms dissipate energy rapidly. In complex systems energy movement through a dissipative system is a process where one thing effects another in a chain of events until all the energy escapes. The human body is an example. Food energy is input at one end of a metabolic chemical process –a sequence or cascade of reactions- while body heat and physical exertions drain it away. Such processes can behave chaotically. Dissipative systems which implement a process constitute a whole dimension of system behavior I've not time to explore here.

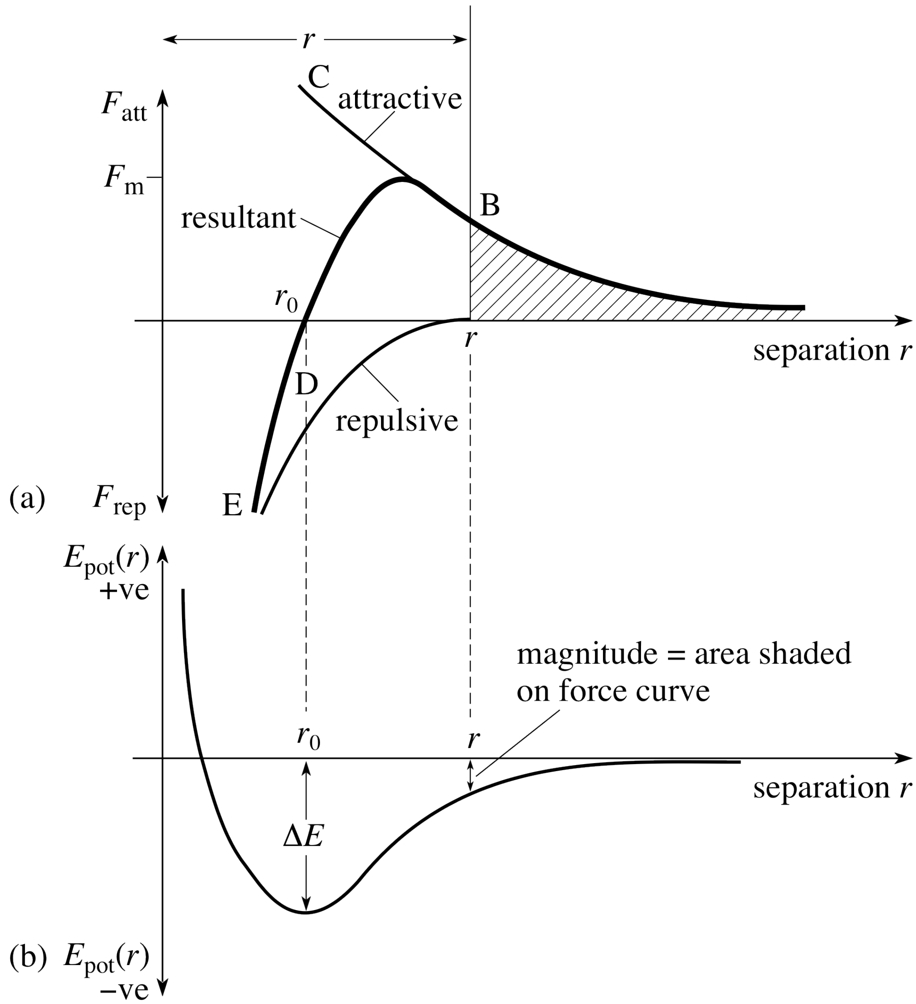
2.2 Forces and behavior:

Force strength: There are four basic forces in nature:

> **Gravity** Weak but extends over cosmic distances
> **Electromagnetic** Fairly strong. Source of magnetic attraction and also source of electrostatic forces where opposite charges attract and like charges repel. Works at the atomic and molecular level to bind atoms into molecules and hold solids together.
> **Nuclear Strong Force** Extremely strong. Holds atomic nuclei together. Powers hydrogen fusion bombs. Acts only over a very short distance within atomic nuclei.
> **Weak Nuclear Force** Not relevant for our purposes.

Only gravity, electrostatic, and centrifugal forces are relevant in systems discussed in this book. Centrifugal force is not one of nature's fundamental forces however.

Balance between attracting and repelling forces: The physics associated with intermolecular forces, namely the forces that hold molecules together to form liquids and solids, underlay the simple diagrams to follow in Figure 111. The image below shows how the "resultant" or net force between two molecules is the summation of attractive electrostatic forces and repulsive electrostatic forces, each of which varies with the distance between the molecules. Electrostatic forces boil down to "like charges repel" and "opposite charges attract". The physics behind these forces gets complex but apparently the repulsing force gets extremely strong when the electron clouds surrounding each atom get too close. http://hyperphysics.phy-astr.gsu.edu/hbase/molecule/paulirep.html. These repulsing forces are what makes it very difficult to crush solid materials or compress liquids.

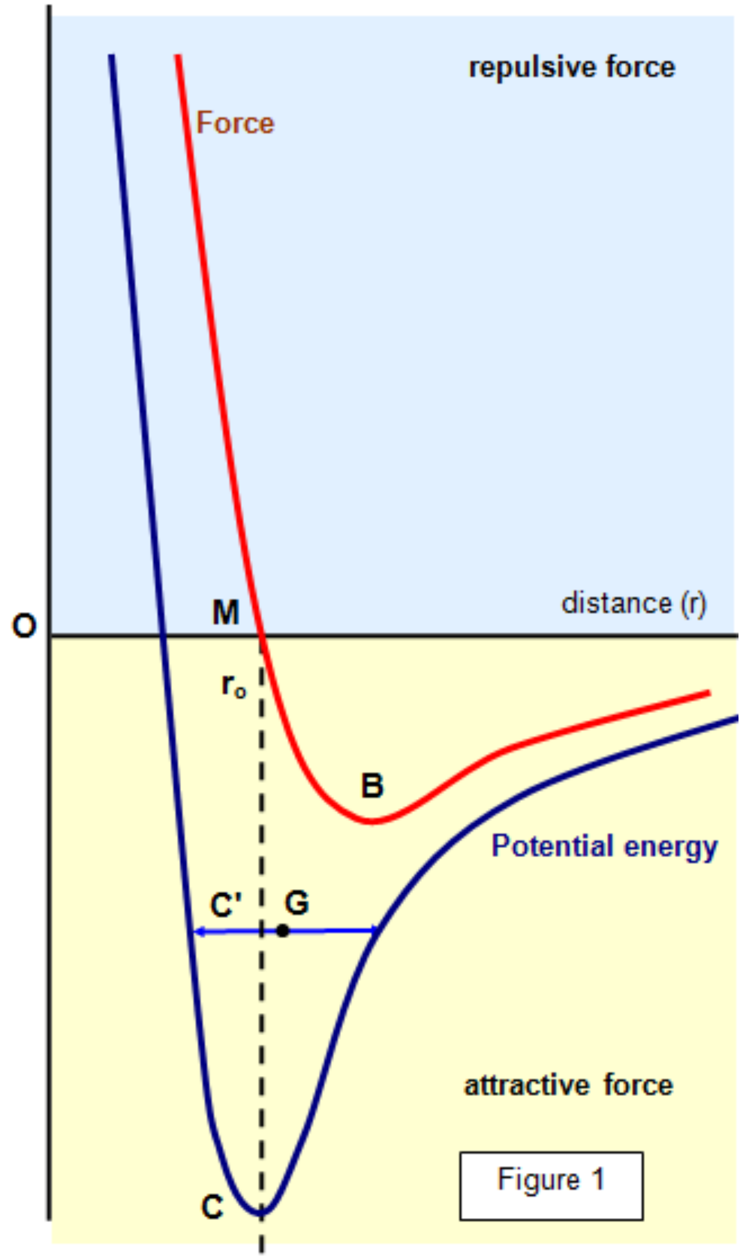


The red line in image below shows one again how the net force between two molecules varies with the distance between them.
http://www.schoolphysics.co.uk/age16-19/Thermal%20physics/Heat%20energy/text/Forces_between_molecules/index.html

At the equilibrium distance "M" the net force is zero. This is also the lowest energy configuration, meaning it takes an input of energy to either push the molecules closer together or pull them further apart. On the other hand if the molecules start on either side of the equilibrium they give off energy –in an exothermic reaction- as they approach that equilibrium. If heat is applied to the equilibrium configuration the distance between the molecules would begin to oscillate around the "M" position. In the real-world there is always some heat so molecules always occillate internally, gently at low temperature, violently at high temperature. The potential energy curve can also be visualized as a ramp or well. If a ball were at the bottom it would take energy to roll it up on either side and create a pendulum like oscillation.

The shape of this red curve below should be remembered because its fundamental to understanding the dynamics of all systems where separate parts are attracted by one type of force but repelled if they get too close by another type of force. The result is they stay a certain distance apart –or oscillate about it- and form a structure or configuration. This is true whether the forces between the parts are springs, electrostatic, gravitational, or the nuclear strong force. I think a similar situation is found in all other types of systems: ecological, economic, social, etc. There is some force that binds parts into a system and another which keeps them from merging entirely. Sometimes it probably amounts to a balance of power or force between competing economic and political organizations. The compromise they reach amounts to an equilibrium. The system they form in that equilibrium is like a giant molecule. A societal molecule so to speak.

The four fundamental forces in nature can act in sequence as masses get closer to each other. At long distances gravity pulls things together until they form a solid ball at which time repulsing intermolecular forces take over and keep the ball from being crushed further. Planets are good examples. If the ball of matter is large enough -like a burnt-out star of sufficient size- gravity will overwhelm the repulsive intermolecular forces and crush the star into a neutron star. Something called "Degenerate Neutron Pressure" will resist further collapse at that point. However if the star was large enough no force can overcome gravity and the dead star will collapse into a black hole.
http://minerva.union.edu/vianil/web_stuff2/Election_and_Neutron_Pressure.htm



Behavior: Behavior is what a system does, how it changes over time. Change is measured in terms of the position or speed of the parts, temperature, pressure, or some other attribute that varies over time. In societal systems it might be group membership, organizational structure, size, territory, revenue, market share, policy position, projects undertaken, and so forth. All these are called variables.

When a spring/mass system is in equilibrium all the springs are relaxed and the parts aren't moving. When one part is disturbed, that is pulled aside by some outside force, the springs are stretched. When the part is released the springs try to pull it back to its equilibrium position. However the part doesn't stop there but rather overshoots and then gets pulled back the other way. Thus the part oscillates back and forth, until and unless friction slows it to a stop. A main premise of this book is that most real-world systems behave this way, be they mechanical like the solar system, chemical like living organisms, or ecological, economic, or political. The actions of one part affect the parts it has relationships with. Of course real world systems of practical interest are not connected by springs, but since parts do affect each other there must be some force or force equivalent linking them. Using springs is a convenient way to represent this connecting force.

Figure 111 takes a closer look at this. For a system to exist there need to be two types of forces; one pulling the parts together so they make a system and other keeping them some distance apart so they remain separate parts. The attracting force in solar systems is gravity and the attracting force in molecules and solid objects is some version of electrostatic or electromagnetic force. Both are non-linear in that their strength falls off exponentially with distance. The spring mass systems obviously are held together by springs, which at the molecular level also involve electromagnetic forces. However at the macro level most springs are linear meaning twice as much force stretches them twice as much. I won't get into repelling forces now except to say they exist, although not always in the form expected. For instance centrifugal force is not one of natures basic forces but it does balance gravity to keep planets in orbit.

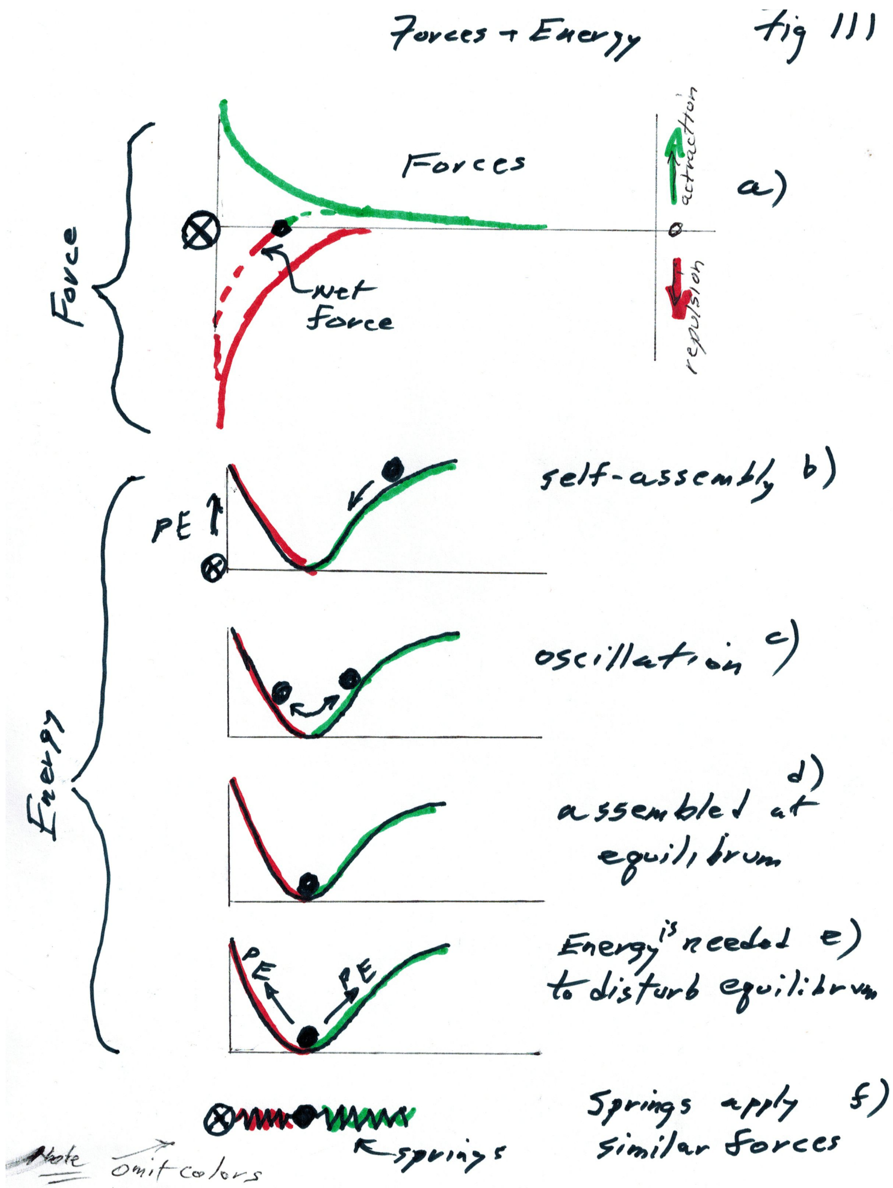


Figure 111a illustrates the interplay of attracting and repelling forces. The large ball with an x through it at left is one part and the small black ball is the other part. The green line represents a long range attracting force, which, like gravity, gets weaker the further apart these two bodies are. The red line represents a repelling force that's much stronger at close distance but falls off rapidly with distance. If this kind of repelling force didn't exist solid bodies like earth would be crushed by gravity into a tiny spec. And as black holes testify if gravity is strong enough burnt-out stars will be crushed to miniscule "singularities". The dotted line is the sum of these attracting and repelling forces and thus the net force on a body. At one particular distance the net force is zero and a body placed there will remain in equilibrium. If it gets closer it will be pushed out. If it is further out the net force pulls it in.

The next four diagrams look at things from an energy perspective, remembering that to push against a force takes energy.

Figure 111"b" thru "e" use an easy to understand ramp analogy. If a part is out on the green side of the ramp it will roll in and downhill toward the low point. If its on the red repelling side it will roll out. Here the ball is shown rolling in. This is the essence of systems self-assembly. Parts that start far apart are pulled together until they reach an equilibrium distance apart. The big-bang blasted all the matter in the universe far apart and –in localized "gravity bound" areas like galactic clusters-gravity has been pulling it back together ever since. At some point gravity will be balanced by a repelling force and inward movement will stop. Again we make an exception for black holes.

Figure 111c shows that if the ball is pulled aside and then released it will roll down one side and up the other. Without friction this will result in endless back and for the oscillation. Arguably its how almost all systems actually behave to one degree or another. Its key to note that when the ball is high-up is has potential energy or PE. The force exerted on it at that point accelerates it toward the equilibrium pint thus converting the PE to kinetic energy as the ball speeds up. This is the essential core reason why systems oscillate.

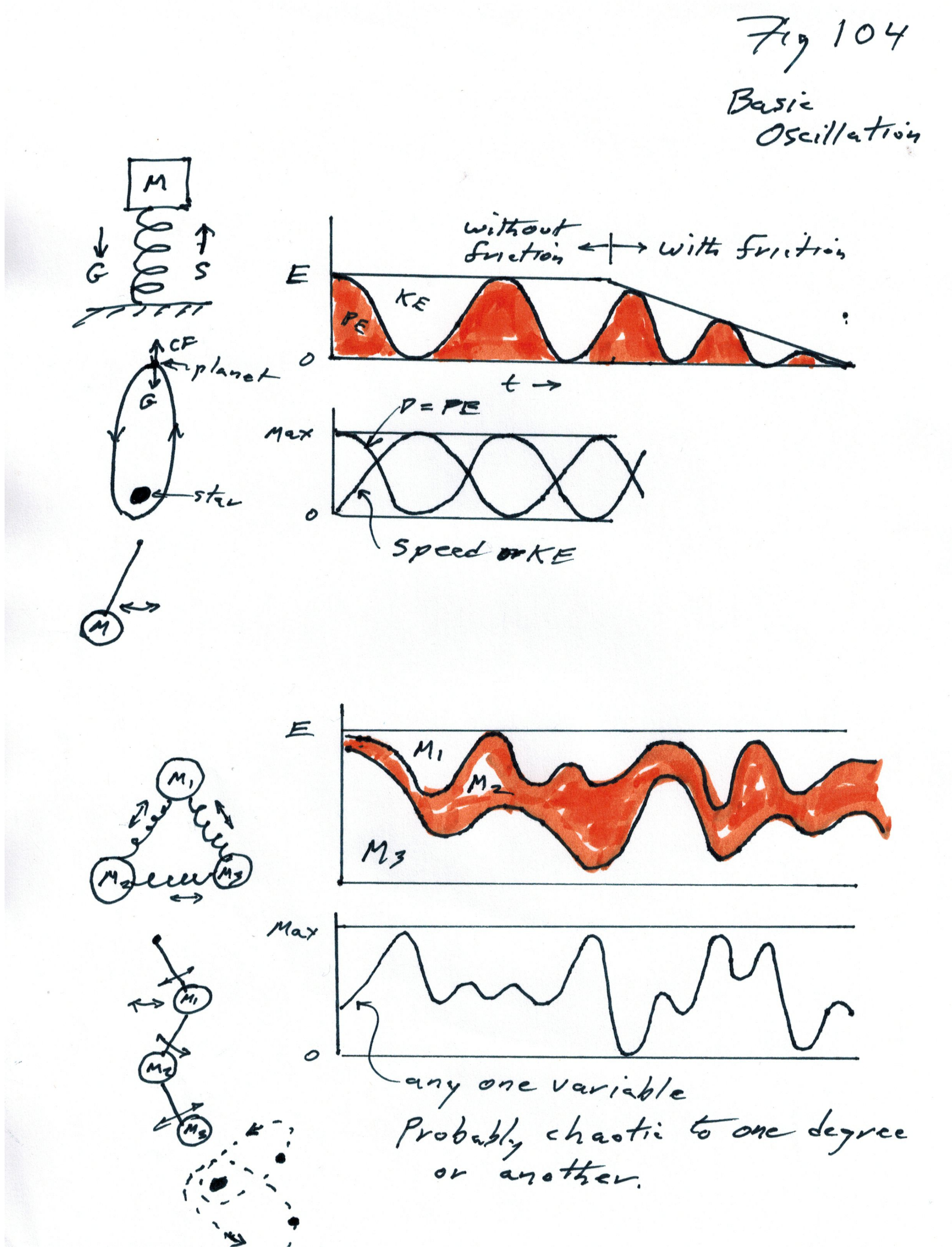
Figure 111d shows the body at a distance where the forces are balanced and thus don't seek to move it. Its at a motionless equilibrium. Its fair to say all systems tend to evolve toward an equilibrium configuration where all forces are balanced and nothing changes or moves. However not all equilibriums are equal. There are relatively weak of unstable local equilibriums and if a system in one is disturbed enough it might snap into a different and more stable configuration. That would bring it to a lower energy level.

Figure 111e shows why it takes energy to move a part once it's in an equilibrium configuration. Its like rolling the ball uphill either way. Same thing when trying to change a system when its in or near equilibrium. In the real world it explains why it takes considerable effort to change molecules, government policy, organizational structure, or start a revolution.

Finally Figure 111f shows the equivalent situation if springs are applying the attracting and repelling forces. Obviously when relaxed they hold things apart some distance, when compressed they become repelling forces and when stretched they act as attracting forces. Thus we can build real physical models and computer models using springs that mimic the behavior of systems where the parts are effected by gravity and other forces. In any system where one part influences another some spring-like force is involved. If the parts are rigidly attached its really the equivalent of very very stiff springs. The systems we study in this chapter mainly involve gravitational or magnetic forces, but some use springs. There is a difference in how they behave because springs apply linear forces whereas gravity and magnetic forces are non-linear. To generalize I often say "spring-like forces" when I’m not sure whether they are linear like simple springs or non-linear like gravity.

2.3 Systems and their waveforms

Figure 104 shows that systems with just two parts oscillate differently than systems with three or more parts.



In a two-part system if one part is pulled aside or set in motion it pushes or pulls on the other causing it to move in a sinusoidal oscillation that, without friction or collision, will continue indefinitely. Figure 104 introduces the basic concepts. At top left we see three different two-part systems. The upper has a mass sitting on a spring such that gravity pulls it down while spring pressure pushes it up. When displaced from equilibrium it will bounce up and down and any variable will trace a perfect sine wave as shown on right. Without friction each wave will be the same as those before and future behavior can easily be predicted. If the weight is lifted it will introduce potential energy into the system just as lifting a weight off the floor would. If it is pushed down it will gain spring potential energy. Thus at time zero on left the system has some level of total potential energy "E". When the weight is released the mass will accelerate converting potential energy (PE) into kinetic energy (KE). Without friction the law of conservation of energy means the total energy must stay constant but it can exchange between different forms, namely PE or KE. The plot of any variable in this system whether its PE, KE, speed, or the displacement of the mass will form a sign wave. The plot shows that as displacement or potential energy increases, speed or kinetic energy falls. The values must always stay in an envelope as determined by the total energy in the system. We will see that this applies whether the motion is periodic as it is here or chaotic. Variables do vary but only within bounds, even when that variation is chaotic.

The lower part of Figure 104 shows waveforms associated with systems having three or more parts. The parts don't oscillate with a simple sinusoidal motion. Rather their movements are more complex, possibly chaotic, because each disturbs all the others.

The key point about systems with three or more parts is that the parts rarely move in a perfectly repeatable or periodic manner as the chart attempts to illustrate. The peaks and valleys in their waveform are constantly changing. In addition the energy in the system moves from part to part because the pushing part slows down and transfers energy to the part being pushed and accelerated. Figure 104 attempts to show how the energy in each part changes over time. And that it usually changes randomly or chaotically.

2.4 All systems are sub-systems, and big dominates small

These two points were already mentioned in Chapter 1 but inserted here just for a reminder.

As described in Chapter 1 no system exists in total isolation. Computer models often assume they do but it's not true in the real world. The behavior of a real world system is always affected to some degree by events in its environment, including the behavior of other systems to which it is coupled, weakly or strongly.

There is a hierarchy of systems ranging from the very powerful to the very weak. The most powerful system of practical concern is the solar system whose behavior affects almost everything on earth, but it can't be argued that what happens <u>on</u> earth has any noticeable effect on earths orbit. "Small" systems, even ones as large as the economy ride atop and are influenced by larger underlying systems like ocean current circulation and climate, which in turn ride atop and are thus influenced in the long term by changes in earths orbit caused by interaction with other planets.

The big parts within a system affect the small parts more than vice versa. For instance Jupiter's behavior gravitationally affects earth far, far more than the reverse. And clearly the suns behavior, especially solar storms, affects earth but its hard to imagine how earth could affect the sun in any meaningful way. Government has far more affect on an individual corporation than the reverse. One reason to mention this is that the parts in toy systems like the double pendulum tend to be more or less the same size. Not different by an order(s) of magnitude.

2.5 Fluids and gases as systems

This book deals almost entirely with mechanical systems comprised of a few discrete parts connected by spring-like forces into some spatial configuration like a double pendulum, solar system, or molecule. However systems can be made from fluids and gases. We refer to circulating ocean currents as systems as well as the atmosphere. These systems behave chaotically to a greater or lesser extent and will be referenced from time to time but not explored in depth.

Its not easy to see how the same physical laws apply to tiny mechanical systems with a few parts and also to fluids and gases with untold trillions of parts, but it seems that a fluid or gas is essentially a huge spring/mass system where the vibration and movement of each molecule effects every other. If we look just at the micro scale where electromagnetic forces govern the vibration within molecules we tend to overlook the gravitational forces that also affect groups of molecules at the macro level. Thus when a molecule of water is heated the intra-molecular springs vibrate harder making the molecule effectively larger, and thus less dense. Gravity acts on a blob of less-dense water making it rise up. Thus on the macro scale we see convection currents in the ocean and atmosphere, and consider their configurations as systems in their own right. We have a hierarchy of systems from the very small to the very large and a behavior at each level. I haven't time to really focus on the behavior of fluidic systems in this book.

2.6 Energy

Conservation of: If there is no friction or other way to throw off energy then the total level of energy in a system remains constant. The law about conservation of energy requires that it all stay within the system although it can oscillate between its potential and kinetic forms and move from place to place or part to part. Real-world systems almost always shed energy thru friction, which turns to heat, and then radiates away. Gravity waves also shed energy from planetary or stellar systems, albeit very slowly. Almost all the simulations I discuss in this book conserve energy so they contain the same total energy throughout. This is realistic enough for our purposes.

System as energy storage device: In many natural systems made with parts having mass–as opposed to societal systems- energy is stored in the system when parts are moving and thus have kinetic energy, or when the spring-like forces are either stretched or compressed. This gives them potential energy. A grapefruit sitting on the counter has potential energy because lifting it there stretched its gravitational bond with earth.

Energy in dissipative systems: Systems are either conservative or dissipative. In a conservative system there is no friction or other way for energy to bleed off so the total energy in the system remains constant. Dissipative systems dissipate energy as they move. Energy is added to the Lorenz waterwheel by pouring water in at the top. It dissipates or bleeds off when the water leaks from the cups at a lower level. In the Rayleigh-Benard convection cells its added as heat along the bottom and dissipated at the cooler top. Assuming the energy bleeds off at the same rate its added I believe that there is still a certain fixed amount of energy in a dissipative systems and the parts must move so as to express or contain it.

Energy changes form and moves within the system: The back and forth conversion of energy from its potential form to its kinetic form is well recognized but strangely enough I've not seen the fact it moves from place to place or part to part mentioned in the literature. I discuss this a lot later but for now suffice it to say that in a double pendulum the arms push and pull on each other. The pushing arm gives some of its energy to the one being pushed so energy moves, actually oscillates between the different parts.

2.7 Defining chaos

C**asual definition:** The casual definition of chaos is that utter confusion exists and anything can happen. Scientists have a very different definition. To them, chaos is a random looking oscillation that stays within bounds and follows the laws of physics.

Scientific definition: According to Strogatz there is no universally accepted definition of chaos. however he offers the following.

> "Chaos is aperiodic long-term behavior in a deterministic system that exhibits sensitive dependence on initial conditions" (Ca2, p.323)

Note the word "aperiodic". During my early research I fell victim to thinking that any behavior that's aperiodic is by definition chaotic, while ignoring his point that it also needs to show SDIC. Both tests should be applied, and I've since done so.

Here is what Wikipedia says by way of definition:

> "Although no universally accepted mathematical definition of chaos exists, a commonly used definition originally formulated by Robert L. Devaney says that, for a dynamical system to be classified as chaotic, it must have these properties:[12]
>
> 1. it must be sensitive to initial conditions
> 2. it must be topologically mixing
> 3. it must have dense periodic orbits
>
> In some cases, the last two properties in the above have been shown to actually imply sensitivity to initial conditions.[13][14] In these cases, while it is often the most practically significant property, "sensitivity to initial conditions" need not be stated in the definition." https://en.wikipedia.org/wiki/Chaos_theory

I don't clearly understand what's meant by topologically mixing or dense periodic orbits but I think they refer to phase space plots and that the double pendulum and other toy systems discussed in this book satisfy those criteria.

Here's another from Math Insight:

> "A dynamical system exhibits chaos if it has solutions that appear to be quite random and the solutions exhibit sensitive dependence on initial conditions." http://mathinsight.org/definition/chaos

One of the best plain English descriptions of chaos I've found. Includes discussion of its definition: http://plato.stanford.edu/entries/chaos/#DefCha

Lyapunov Exponent: The Lyapunov exponent is a widely used test for judging whether a system is chaotic or not. It basically measures how sensitive the system is to small differences in initial conditions or SDIC. In other words it measures how fast the waveforms diverge. You can see this simply by comparing them, but the Lyapunov exponent is a technical measure.

> "The System is said to behave chaotically if the Lyapunov exponent is positive, while a Lyapunov exponent less than or equal to zero denotes non-chaotic behavior." http://psi.nbi.dk/@psi/wiki/The%20Double%20Pendulum/files/projekt_2013-14_RON_EH_BTN.pdf Below is a plot of that exponent for the double pendulum.

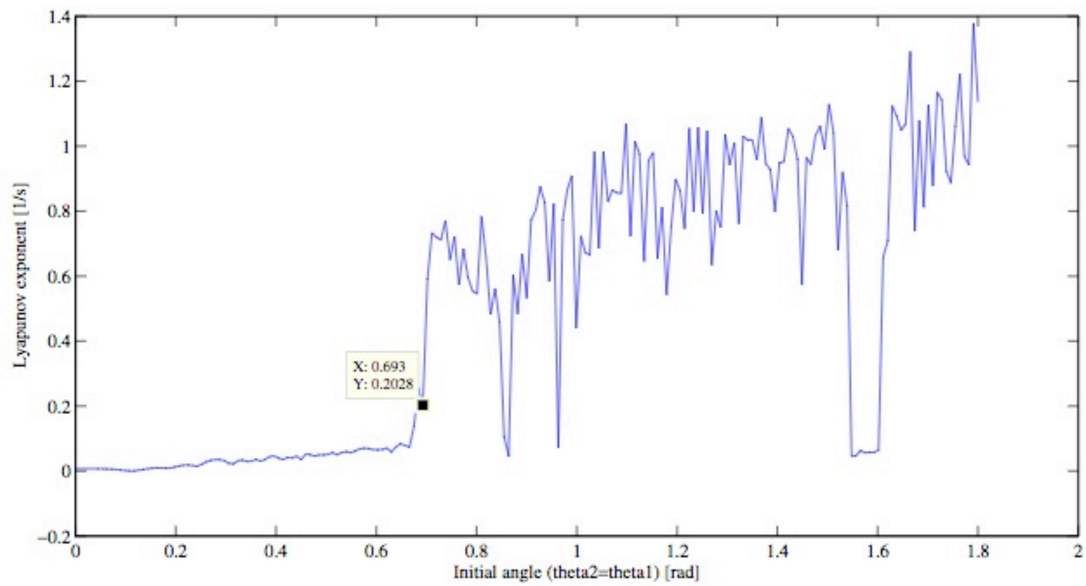


Figure 9: The Lyapunov exponent as a function of initial angle ($\theta_1(0) = \theta_2(0)$, $\dot{\theta}_1(0) = \dot{\theta}_2(0) = 0$)

http://psi.nbi.dk/@psi/wiki/The%20Double%20Pendulum/files/projekt_2013-14_RON_EH_BTN.pdf

According to this test the double pendulum becomes chaotic when the pendulum is released with a1=a2= 0.7 radians (40 degrees). This was apparently computed for a double pendulum with solid arms whereas all my simulations used point masses, which I call bobs. Its not clear that the same angles that mark the threshold to chaos with solid arms apply to point mass bobs. I assumed they didn't.

One problem with using the Lyapunov exponent is that one must know the equations of motion for the system and enough math to compute it.

The SDIC test: Its true and very easy to demonstrate –assuming you have the right computer models- that toy system exhibit sensitive dependence on initial conditions or SDIC. SDIC means that a small difference in the initial condition of a chaotic system will magnify over time and make a very large difference in its future condition. Some experimenters have built two identical double pendulums and released them at the same time albeit at slightly different angles to see how the waveforms diverge. Others have done essentially the same thing using computer models. The first figure below shows the waveforms –generated by a computer model- of three double pendulums released at almost the same low 10 degree angle. They differed only by 0.01 radians (one half degree). The wave for each pendulum is plotted in red, blue or green. With 10 degree release they had very little energy in this run. Each block shows a different variable. For instance the upper left plots the angle of the upper arm. It's a bit hard to see but the blue, red and green waveforms differed little in this simulation. At first glance they seem periodic but closer inspection shows the heights of the peaks are slowly drifting. My experiments showed the same thing at low energy. Namely this is not perfectly periodic behavior, rather its quasi-periodic.

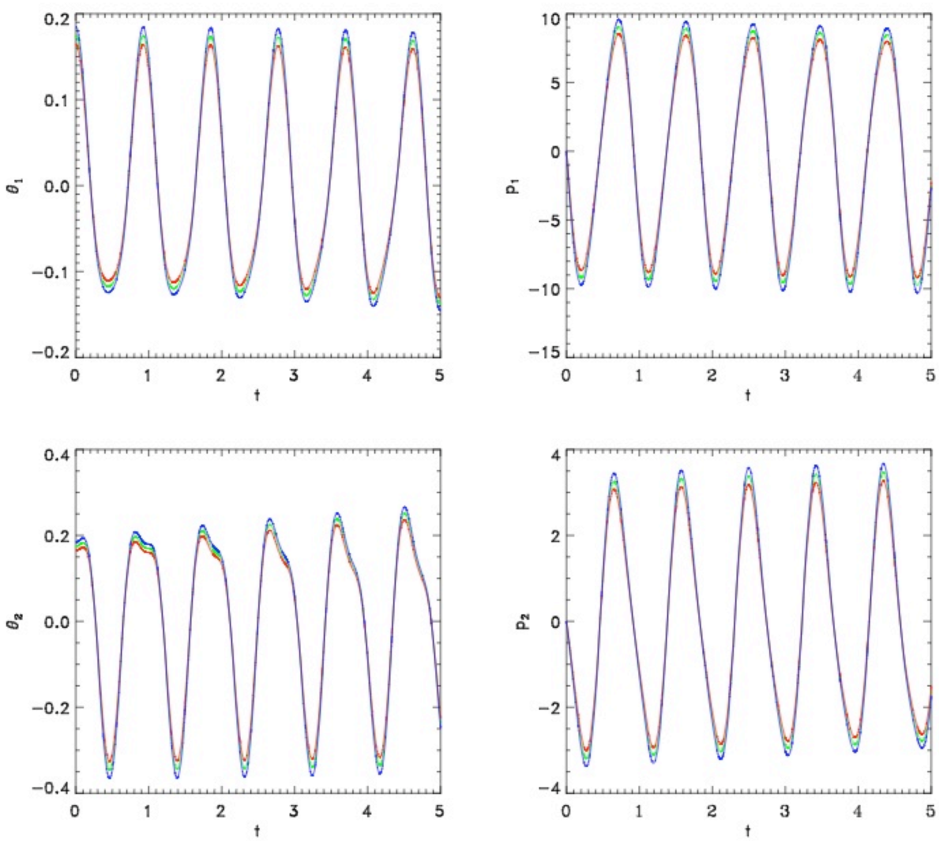
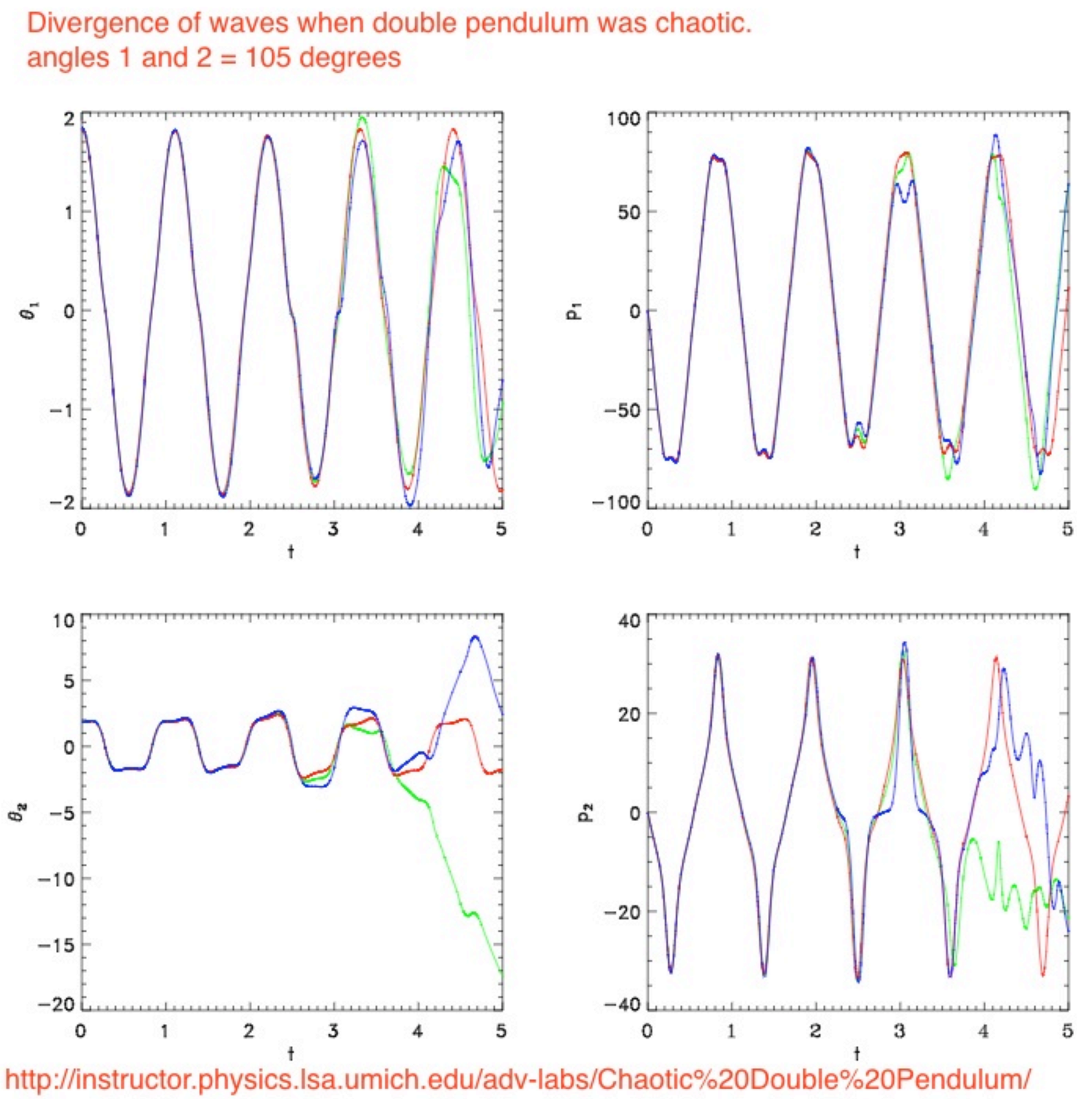


Figure 15. Graphs of $\theta_1(t)$, $p_1(t)$, $\theta_2(t)$, $p_2(t)$ for $\theta_1(0) = 10°$, $\theta_2(0) = 10°$. The red, green and blue lines show the behavior for three initial conditions for θ_1 and θ_2 that vary by 0.01 radians.

http://instructor.physics.lsa.umich.edu/adv-labs/Chaotic%20Double%20Pendulum/Pendulum_2010_04_12.pdf

The image below shows how the red, green and blue waveforms diverge then the arms are released at a much higher 105 degree angle giving the system considerably more energy and making it chaotic.

These waveforms show why the long-term behavior of chaotic systems can't be predicted. Which line is right, the red one, the blue one, or the green one? On the other hand the three lines lie close tighter for a while indicating that short-term forecasts are possible with a good model. This is the reason its relatively easy to predict whether it will rain three days from now, but not 10 or more days out. We can of course cite probabilities based on historic records.



Eyeball test: Unfortunately I didn't have access to a model that could plot waveforms for two or more double pendulums at the same time so I was unable to make plots like those above and use them to say if a particular run was chaotic or not. I was however able to make two separate runs with slightly different initial conditions and compare them visually. I used that technique extensively in my analysis of the double pendulum. Details appear in Chapters 7,8 and 9.

Systems behavior including chaotic behavior is deterministic. This means that the way the system is today is the sole determinant of how it will evolve in the future. Starting with its current situation the way it will change or evolve will obeys the laws of physics. Nothing in the behavior of these systems happens by chance or luck. Waveforms do appear to vary randomly, and they are often described that way, but its just physics doing its thing.

2.8 Videos and references about chaos

A nice 4-minute introduction to chaos by James Gleich is found at:
http://www.clausewitz.com/mobile/chaosdemos.htm#DblPend

Compelling lab demo using rubber sheet to show solar system dynamics:
https://www.youtube.com/watch?v=MTY1Kje0yLg

Sophisticated discussion in plain English about the definition of chaos with extensive references at: http://plato.stanford.edu/entries/chaos/...

A readable discussion of chaos included doffing oscillator. Phase space, etc.:
https://books.google.com/books?id=l2E4ciBQ9qEC&pg=PA117&lpg=PA117&dq=chaos+spring+mass&source=bl&ots=7HJuhOrR4X&sig=n2i2PQJO5HLIQhbDdTFLDz0ko1M&hl=en&sa=X&ved=0ahUKEwiXyoyWtdfJAhUQ0mMKHV7zBd4Q6AEINDAD#v=onepage&q=chaos%20spring%20mass&f=false

This is good on climate and chaos:
http://www.realclimate.org/index.php/archives/2005/11/chaos-and-climate/ and so is this: https://www.aip.org/history/climate/chaos.htm

2.9 Tools and techniques for analysis

There are simple mechanical and electrical systems whose real-world behavior can be observed, and watching them is instructive. However computer simulation models are far more useful since one can accurately vary certain parameters and make plots showing how the variables change in response. Several Java models easily obtained on the web have been used by the author to great benefit. A model of the double pendulum created by Dr. Dooling has been especially helpful.
http://www2.uncp.edu/home/dooling/applets/double_pen.files/tom/models/doublepen.html. Or
http://www2.uncp.edu/home/dooling/applets/double_pen.files/tom/models/doublepen.html

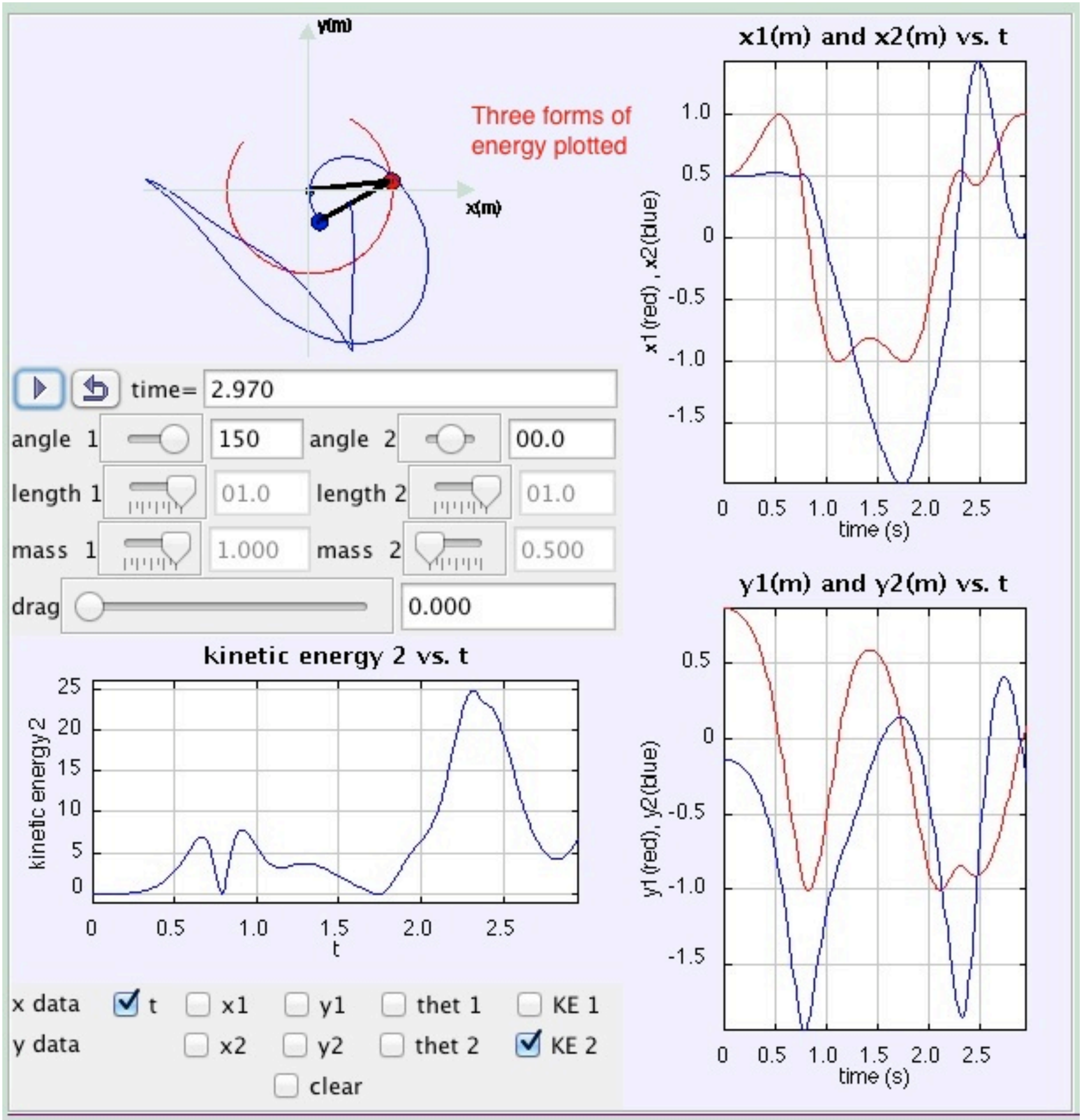
With models one can plot how variables changes over time but sometimes its more instructive to show how one variable is changing relative to how another. If there are two variables the value of one is plotted on the X axis and the other on the Y axis so we get a 2-dimensional plot. Three variables produce a 3-D plot. These are called "phase space" portraits or plots. If there are more than three variables the phase space plot would require 4 or more dimensions. That's impossible; however even plotting two or three is often sufficient. Its often easier to see how periodic, quasi-periodic or chaotic a system is by making phase space plots that it is by looking at the waveforms. The value of all the variables at any instant shows up as a dot in phase space but as the variables change the dot moves leaving a trace or trajectory behind it. If the dot traces the same pattern time after time the system is oscillating periodically. If not its either quasi-periodic or chaotic.

I highly recommend spending some time with these simulation models as nothing gives a feel for a systems behavior better than watching it in action. They are often written in Java and easily downloaded from the web after getting your security settings to accept them.

The screenshot below shows the user interface to Dr. Dooling's model of the double pendulum. I used it extensively. Users can adjust any of the parameters. When both bobs hang motionless and straight down there is zero energy in the system. Before initiating a run the bob or bobs are lifted to some angle in order to insert potential energy into the system. Usually I just varied angle 1 to see how the system behaved at different energy levels.

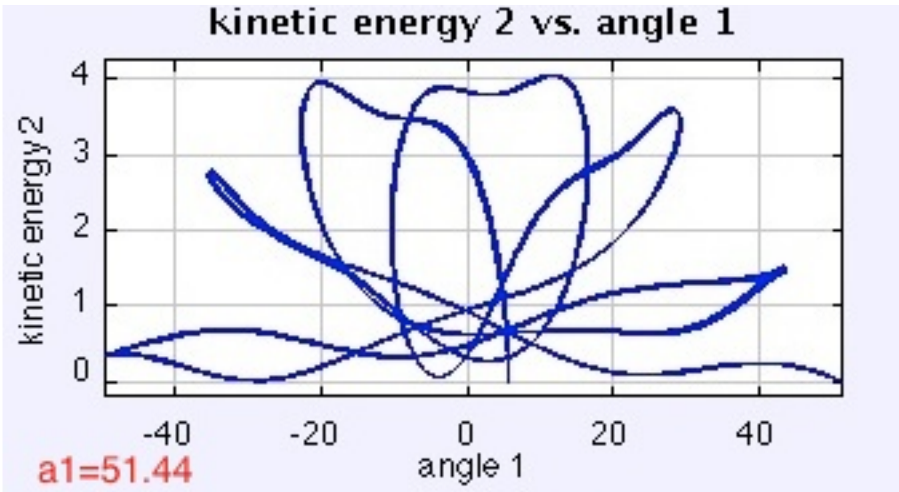
These screenshots show the user interface and some of the plots that can be generated.

The upper left diagram shows the pendulum bobs and the last few seconds of the trace left behind them as they move. In this case the lower left plot shows a waveform depicting how the kinetic energy of the blue bob changed over time. In this run the red bob was released high so the system had a large amount of energy. This made it chaotic and enabled the blue bob to swing over the top or spin on occasion. Here its just gone over the top. The waveform is highly irregular but we haven't watched it long enough or seen enough other data yet to conclude whether its periodic, quasi-periodic or chaotic.

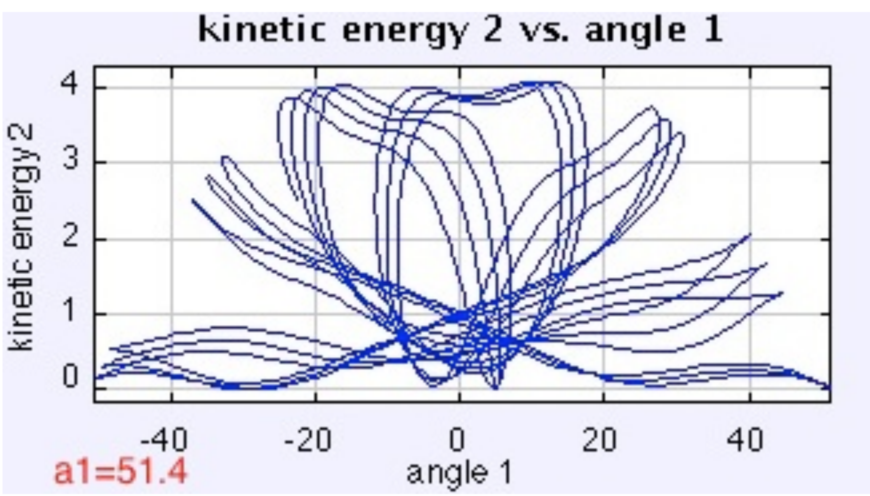


Usually its easier to determine if a system is behaving perfectly periodically, quasi-periodically, or chaotically by plotting one variable against another as shown in the screenshots below. This produces a 2-D view into what is really a 4-D "phase space portrait" because the double pendulum has four variables PE1, PE2, KE1 and KE2.

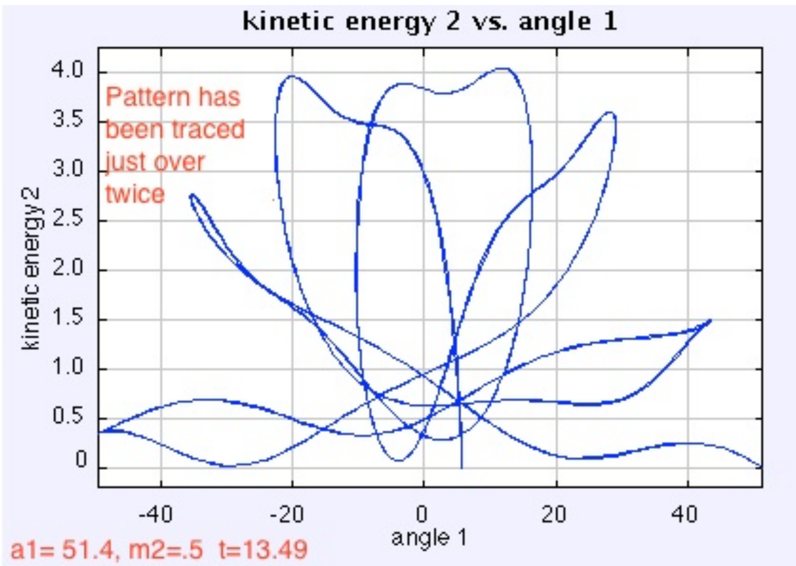
The first screenshot below the 2-D or partial phase space plot for a system that was oscillating perfectly periodically or very close to same. The trace goes round and round retracing exactly the same pattern time after time, one atop the other. Given where all the other parameters –line bob mass and arm length- were set the release angle a1 had to be exactly right to get perfectly periodic operation. In this case it was 51.44 degrees. This value was found by trial and error starting with a run that just seemed close.

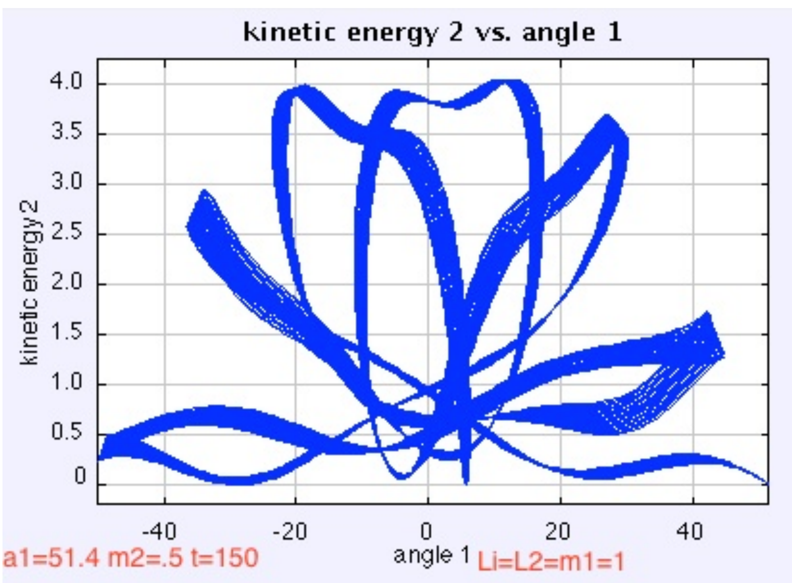


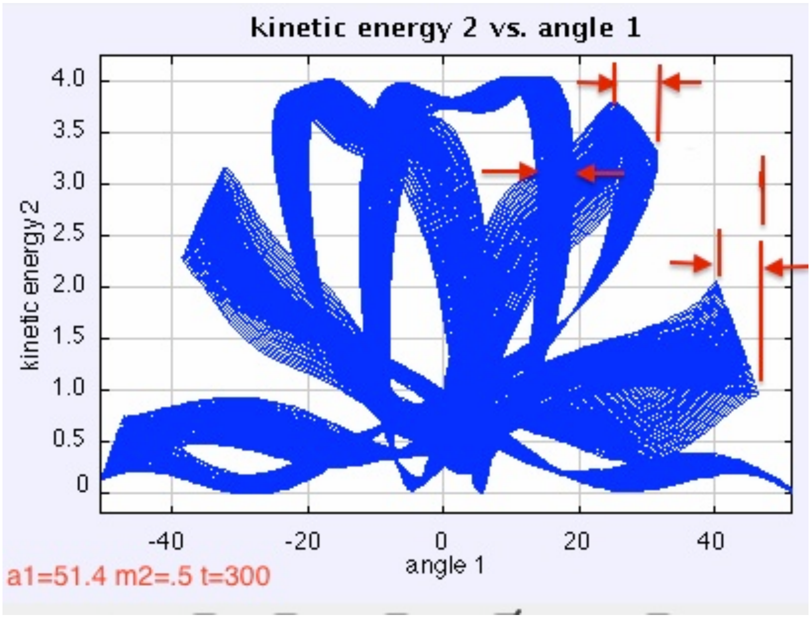
Below we show what quasi-periodic behavior looks like. The trace makes the same general pattern over and over –it was stopped for this screenshot after four times- but the values are slightly different each time so the patterns are offset. These lines will continue to drift until they fill-in the entire envelope. The size of the envelope is limited by the energy in the system. In this case angle 1 can never exceed the 51 degrees it was released at. Note that angle a1 was almost the same as the angle that made the run above perfectly periodic. The perfectly periodic run was found by fine tuning angle 1 by trial and error.



The following three screenshots from another run show how the values drift over time during quasi-periodic operation. The first was taken about 13 seconds into the run, the second was taken after 150 seconds and the last after 300 seconds. I judged this as quasi-periodic because it was obvious that the general pattern of behavior repeated time after time but since the patterns did not overly precisely this was not perfectly periodic behavior. Other more casual observers have apparently called this behavior "periodic", which is a loose interpretation of that term.

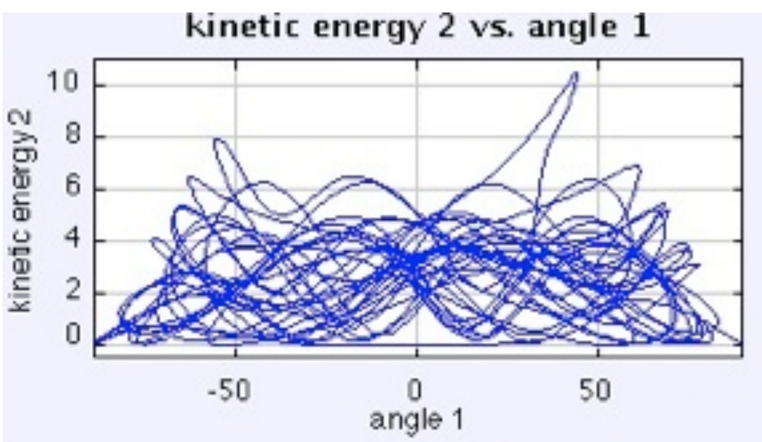






If the above run had run long enough the trace would have probably filled in the entire envelope as it did in other quasi-periodic runs. The red lines show the range over which variable a1 changed. I usually say the values or patterns drift. Its obvious that some amount of prediction is possible when the system is operating quasi-periodically.

Below is an example of chaotic behavior. The trace follows a random route. There is no consistent pattern. If let run longer these lines would also fill-in the entire envelope.



You should now have some idea how these tools are applied to better understand the behavior of this system. We discuss the useful insights later.

If one variable in a system is changing it means all the others are changing in similar manner. In other words we could plot angle 1, angle 2, ke1 or ke2 and they would all have similar waveforms over time. Its handy to know that when viewing simulations. Having said that, plotting two variables in a partial phase space plot makes it easier to determine if a system is perfectly periodic, quasi-periodic or chaotic.

The stage is now set to have a closer look. After showing pictures or linking to videos of some dynamic systems in Chapter 3, Chapters 4 and 5 will follow by listing what I feel are the most important findings about systems behavior that I've become aware of. Each finding or conclusion will include plots, screenshots, or other data to support it so its not just speculation.

Chapters 6,7,8, 9 and 10 describe in detail simulation runs I've made in order to discover how the double pendulum and magnetic pendulum behave.

Chapters 11 and 12 attempt to relate what we know about toy systems to several large real-world systems. Experts like Strogatz write that the application of chaos theory to complex systems is largely "unexplored territory". (Ca2, p.10)

Making that jump involves some speculation on my part. That's partly because I haven't found simulation models to help bridge the gap. In other words models that have more parts than the small toy systems studied so far and/or are designed for serious analysis. For instance the spring/mass simulations I've found with about 6 parts don't produce adequate waveform and phase space plots and/or they don't allow initial conditions to be set accurately. What I think would be most helpful is a spring/mass model that could handle up to say 20 masses all connected by springs. The user should be able to select the number of masses, insert springs wherever he choses, make the springs linear or non-linear, produce all the requisite plots, be able to plot the force on any mass, track the energy of each mass, be able to test for SDIC, etc. In Chapter 11 I've described such a model in more detail.